Bumps on the Core-Mantle Boundary: Geomagnetic and Gravitational Evidence Revisited [and Discussion]

S. R. C. Malin, R. Hide, K. Lambeck and H. K. Moffatt

Phil. Trans. R. Soc. Lond. A 1982 **306**, 281-289
doi: 10.1098/rsta.1982.0088

Phil. Trans. R. Soc. Lond. A **306**, 281–289 (1982) [281]
Printed in Great Britain

Bumps on the core–mantle boundary: geomagnetic and gravitational evidence revisited

By S. R. C. Malin† and R. Hide, F.R.S.‡
† *Geomagnetism Unit, Institute of Geological Sciences, Murchison House, West Mains Road, Edinburgh EH9 3LA, U.K.*
‡ *Geophysical Fluid Dynamics Laboratory, Meteorological Office (Met O 21), Bracknell, Berkshire RG12 2SZ, U.K.*

Evidence for the presence of undulations or temperature variations, or both, at the core–mantle boundary comes from the remarkably high correlation between the pattern of long-wavelength gravitational anomalies and the corresponding pattern of anomalies in the geomagnetic potential when the latter is rotated in longitude. Since gravitational anomalies originate at or above the core surface and the magnetic anomalies originate within the core, the correlation, if statistically significant, must be a manifestation of processes occurring at the core-mantle interface and has important implications for the structure and dynamics of the Earth's deep interior. Indeed, the discovery of the correlation stemmed from considerations of interactions between motions in the core and mantle and can be regarded as evidence in favour of deep mantle convection. The large value for the correlation coefficient is now accepted but, notwithstanding our published statistical tests to the contrary, opinions are often expressed in the literature that any low-order convergent field might be expected to show a high correlation with gravity when rotated to the optimum position. Here we reiterate that this is not so, and illustrate our argument with a number of randomly generated field models that have correlation coefficients from 0.2 to 0.8.

1. INTRODUCTION

Various lines of geomagnetic, palaeomagnetic and other geophysical evidence, combined with general considerations of the effects of rotation on core motions, led to the introduction in the mid-1960s of the hypothesis that there are bumps on the core–mantle boundary of about a kilometre in vertical amplitude (for references see Hide (1977)). Such bumps would escape detection by seismological methods (for references see Cleary (1981)), but their interaction with core motions would produce temporal and spatial disturbances of the main geomagnetic field. Moreover, owing to the high density contrast at the core–mantle interface, bumps there would contribute significantly to the broad features of the gravitational field at the Earth's surface. The pattern of the distorted magnetic field would be displaced relative to the pattern of bumps and therefore to the pattern of the gravitational field, and if, as is likely, core motions have strong zonal components the displacement would be mainly parallel to latitude circles.

This was the geophysical background and starting point of our investigation of the degree of correlation between the broad patterns of the Earth's gravitational and magnetic fields when one pattern is rotated in longitude relative to the other (Hide & Malin 1970, 1971 *a*, *b*, *c*, 1972). For a numerical test, it seemed obvious to use spherical harmonic coefficients and to truncate these at degree and order $n \leqslant 4$, as used by Hide & Horai (1968), to include only continental-scale features. For the gravitational potential we had a choice of several models, but they were all in close agreement for the low-order coefficients. For the magnetic field the choice was much wider. This was not so much because of the choice of models, but because we had to decide whether we should examine main field or secular variation, potential or force components, and which epoch to use. We also had to decide whether the comparison should be made at the surface or at the core–mantle boundary. With so many possibilities, we could easily have kept testing for correlations until we found one that was significant at the 5 % level. We would expect one of the first 20 correlations to attain this level purely by chance, so it was important to select our options at the outset as this would have a bearing on the significance of the final result. In fact, we started with the simplest of options: we chose only those coefficients of the contemporary geomagnetic field that correspond to the non-zero coefficients of the gravitational field (i.e. the non-dipole field) and did not multiply them by any factors other than that required for full normalization (i.e. comparison of gravitational and magnetic potentials at the surface of the Earth). Despite possible objections on physical grounds – F. J. Lowes later suggested that we should have considered vertical magnetic intensity at the core–mantle boundary – and the fact that we subsequently examined other configurations, it was this first choice that gave us the large correlation coefficient on which we based our claim to have established a significant correlation between anomalies in the gravitational and magnetic fields (Hide & Malin 1970).

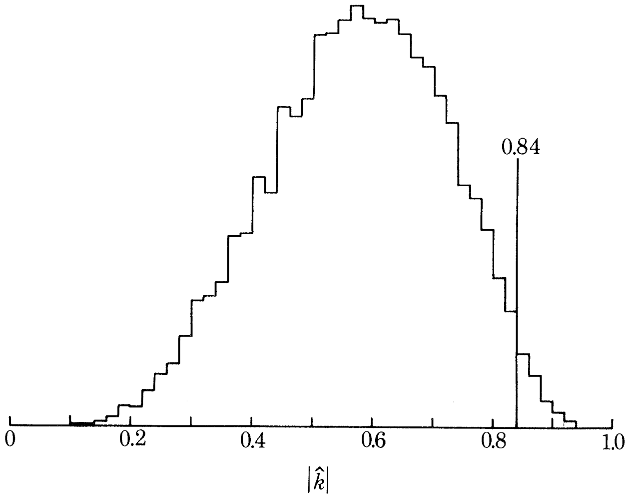

FIGURE 1. Histogram of 10000 values of the modulus of the maximum correlation coefficient, $|\hat{k}|$, obtained from artificial sets of spherical harmonic coefficients. The probability of the observed correlation (0.8366) being equalled or exceeded is only 0.0241 ± 0.0013.

From the outset, we accepted that the magnetic field might be displaced in longitude relative to the gravitational field and we evaluated the angle of displacement $\hat{\lambda}$ that gave the largest coefficient of correlation $\hat{k}$. We were then obliged to devise a test of the statistical significance of $\hat{k}$. Our initial test, based on a method proposed by G. W. Brier (private communication), was not sufficiently robust for our application, and our first claim (Hide & Malin 1970) to have established a significant correlation was challenged by Kahn (1971) and Lowes (1971). Kahn's proposed modification to the test was shown to contain an error (Hide & Malin 1971*a*), and Lowes's argument was based purely on angles and did not take account of the fact that the high correlation is partly due to the similar amplitudes of corresponding gravity and magnetic coefficients. Nevertheless, our test was clearly open to criticism, so we devised a new test based on the Monte Carlo method. For each of 10000 pairs of artificially generated sets of coefficients, we determined $\hat{k}$, the coefficient of correlation that had the greatest modulus, allowing for east–west rotation. The artificial coefficients were random numbers with a Gaussian distribution about zero. They were multiplied by a factor that decreased with increasing order to simulate the observed convergence of the gravity and magnetic coefficients. (The fact that the gravity and magnetic coefficients show similar rates of convergence is a marginal additional argument in favour of the reality of the correlation, but this was disregarded.) As a result of this test, it was found that the probability of obtaining a correlation coefficient with modulus equal to or greater than the observed value of 0.8366 is 0.0241 ± 0.0013 (Hide & Malin 1971*a*), so the correlation is significant at the 2.4 % level. We still consider this to be the best estimate of significance. A histogram showing the distribution of the 10000 values of $|\hat{k}|$ is given in figure 1.

If the correlation implies a physical relation between anomalies in the gravitational and magnetic fields, this must result from interactions at the core–mantle boundary, since the gravitational anomalies can only originate at or above that level, and the continental-scale magnetic anomalies at or below that level.

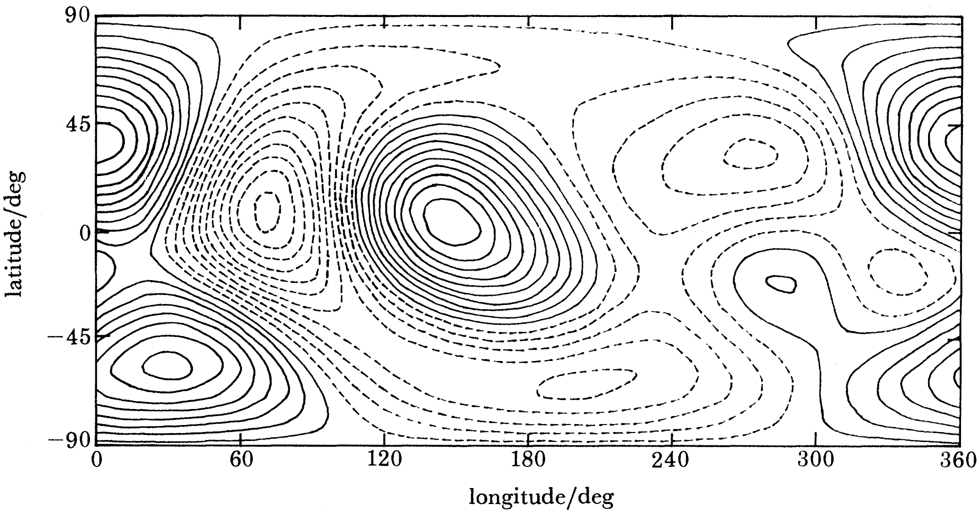

Figure 2. Anomalies in the gravitational potential from the low-order ($n \leqslant 4$) coefficients of a spherical harmonic expansion. The pattern is shown on a *plate carrée* projection bounded at top and bottom by the north and south poles, and on each side by the Greenwich meridian. Solid contours indicate positive values, broken contours negative.

2. Further illustrations

Despite the evidence of the statistical test outlined above and the apparently general acceptance of its validity, we often hear the opinion expressed that it is difficult to visualize a low-order convergent field pattern that will *fail* to show a high correlation with the gravitational potential when suitably rotated. It can be seen from figure 1 that values for $|\hat{k}|$ less than 0.28 are nearly as rare as those that exceeded 0.84, and that the median value for $|\hat{k}|$ is as high as 0.58. Nevertheless, patterns can be found that cover the whole range of correlation coefficients, including the low end, and a selection of these is shown in figures 2–6. Figure 2 shows the pattern of gravity anomalies given by the low-order ($n \leqslant 4$) spherical harmonic coefficients of model CA of Kaula (1966). Figures 3–6 show patterns that have maximum correlation coefficients of 0.2, 0.4, 0.6 and 0.8, respectively, when they are rotated east–west by the appropriate amount. All the patterns are shown at the appropriate orientation for maximum correlation.

The coefficients from which the patterns are synthesized are selected from those generated by the random process described in § 1. The solid contours indicate positive values and the broken contours negative values; clearly, reversing the sign of any of the patterns in figures 3–6 would change the sign of $\hat{k}$ without affecting its modulus.

$|\hat{k}| = 0.2$

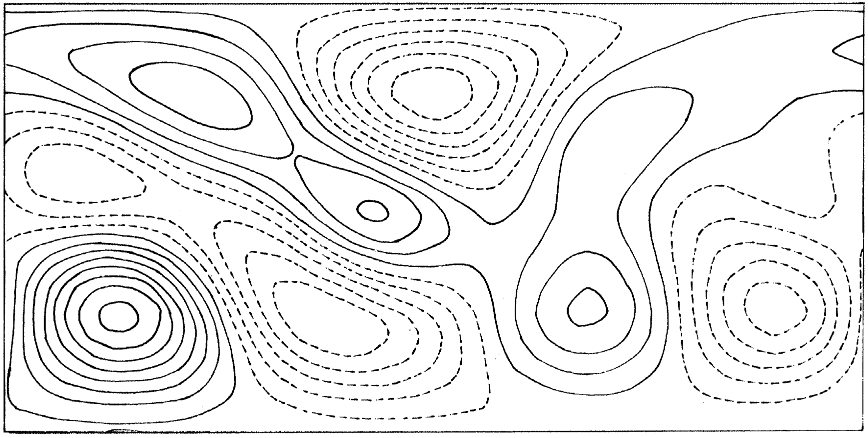

FIGURE 3. A pattern for which the modulus of the correlation coefficient with the gravity potential reaches a maximum of 0.2 at the position shown, when east–west rotation is permitted. Other details are as described in the caption to figure 2.

$|\hat{k}| = 0.4$

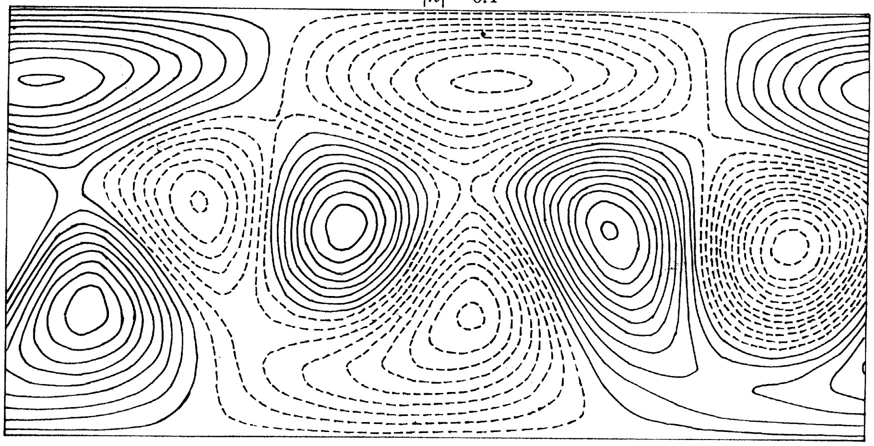

FIGURE 4. A pattern for which the modulus of the correlation coefficient with the gravity potential reaches a maximum of 0.4 at the position shown, when east–west rotation is permitted. Other details are as described in the caption to figure 2.

To illustrate the correlation and its longitudinal displacement, we frequently use a map of gravitational potential covering 720° of longitude (twice round the world) over which we slide a transparent map of magnetic potential. In a spirit of healthy scepticism, K. Lambeck and others (private communications) have asked if the correlation would look equally good if the transparent overlay were placed upside down or back to front! After some calculations we are pleased to announce that the answer is 'No!'. The values for $\hat{k}$ are as follows: 0.59 for a reversal of north and south; 0.78 for a reversal of east and west, and -0.56 for a reversal of both. These values should be compared with 0.84 for the normal orientation.

A more realistic modification of the simple rotation in longitude would be to permit a rotation about a general axis. Acharya (1970) did not believe that this would affect $\hat{k}$, but his argument was shown to be in error by Hide & Malin (1971 *b*). In fact, the optimum pole of rotation is found to be remarkably close to the geographical pole, differing from it by only 2° (Hide & Malin 1971 *b*). This proximity provides some further support for the reasoning upon which we based our initial strategy.

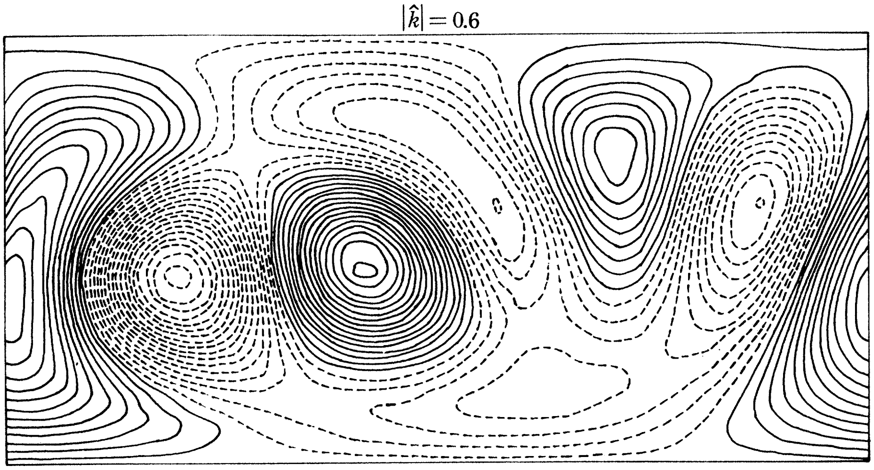

Figure 5. A pattern for which the modulus of the correlation coefficient with the gravity potential reaches a maximum of 0.6 at the position shown, when east–west rotation is permitted. Other details are as described in the caption to figure 2.

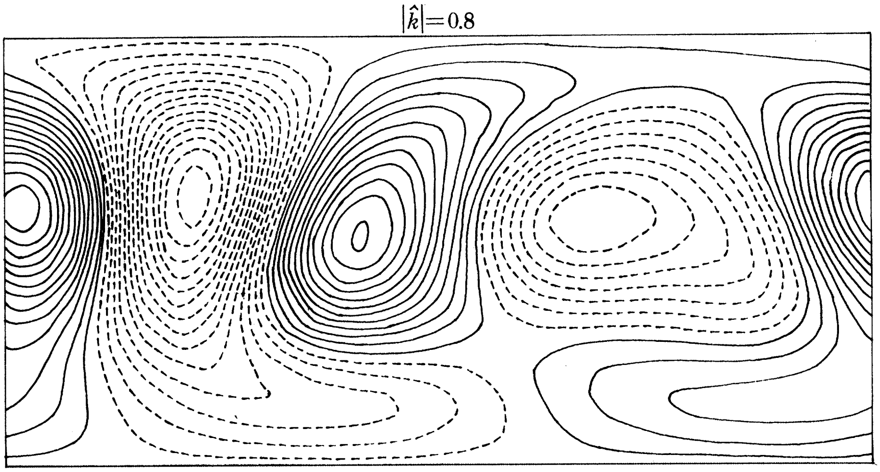

Figure 6. A pattern for which the modulus of the correlation coefficient with the gravity potential reaches a maximum of 0.8 at the position shown, when east–west rotation is permitted. Other details are as described in the caption in figure 2.

Finally, since the magnetic field shows significant changes with time and the gravitational field does not, it might be asked what happens to the correlation and the displacement angle as one goes back in time. This has been investigated by Hide & Malin (1971 *c*, 1972), who show that the correlation coefficient stays close to its present value back to about 1910, but earlier than that it decreases. Since the pre-1910 field models are themselves very unreliable (Barraclough, this symposium), this decrease is probably not significant. The displacement angle also diminishes nearly linearly as one goes back in time, from about 160° at present to about 130° in 1850. This is consistent in both magnitude and direction with the known westward drift of the geomagnetic field.

3. Concluding remarks

The correlation between the Earth's gravitational and magnetic fields has survived detailed scrutiny and criticism over the decade since it was established, and it remains a significant result that provides support for the hypothesis that the core–mantle interface is bumpy. The discovery of the correlation stimulated detailed studies of simple theoretical models of the magnetohydrodynamic interactions between core motions and a bumpy core–mantle interface (see Moffatt & Dillon 1976) that were envisaged in the arguments that lead to the introduction of the hypothesis (Hide 1967), but further work will be needed to provide a truly quantitative interpretation of $\hat{k}$ and $\hat{\lambda}$. We hope to report soon some new work bearing on this problem.

When the hypothesis was first introduced it was coupled with the proposal that the exchange of angular momentum between the core and mantle that is implied by observations of the so-called decade variations in the length of the day is brought about largely by topographic coupling associated with the presence of bumps and not, as had been previously suggested, by electromagnetic coupling involving the leakage of electric currents out of the core into the weakly conducting lower mantle (for references see Lambeck (1980)). Simple theoretical models of the subtle dynamical processes involved in topographic coupling (Hide 1977) are now being investigated and the results so far are encouraging (see Hassan & Eltayeb 1982).

The correlation bears directly on the interpretation of long-wavelength features of the gravitational field in terms of density variations in the mantle (density variations within the liquid core are utterly negligible in this context). The correlation strongly implies that density variations at the core–mantle interface make a significant contribution. It also implies that long-wavelength density variations spread throughout the depth of the mantle are either so small as to produce comparatively small effects on the gravitational field or comparatively large but well correlated with the shape of the core–mantle interface. The latter possibility is consistent with the idea that convection occurs in the deep mantle (for references see Elsasser *et al.* (1977)), where it would provide the mechanical stresses required to maintain bumps on the core–mantle interface against the tendency for gravity to smooth them out (see Hide 1970; Jones 1977). But it is inconsistent with those models that seek to account for the long-wavelength features of the Earth's gravitational fields in terms of density inhomogeneities concentrated in the upper mantle (for references see Phillips & Lambeck (1980) and Rubincam (1981)). There are encouraging signs that seismological studies of lateral density variations in the mantle (see, for example, Dziewonski *et al.* 1977) might in due course provide independent evidence bearing on this aspect of the geophysical interpretation of the correlation between the Earth's gravitational and magnetic fields.

This paper is published with the permission of the Director of the Institute of Geological Sciences (N.E.R.C.) and the Director-General of the Meteorological Office.

REFERENCES (Malin & Hide)

Acharya, H. K. 1970 *Nature, Lond.* **228**, 758–759.

Cleary, J. 1981 *Physics Earth planet. Inter.* **26**, 266–267.

Dziewonski, A. M., Hager, B. H. & O'Connell, R. J. 1977 *J. geophys. Res.* **82**, 239–255.

Elsasser, W. M., Olson, P. & Marsh, B. D. 1979 *J. geophys. Res.* **84**, 147–155.

Hassan, M. H. A. & Eltayeb, I. A. 1982 (Submitted for publication.)

Hide, R. 1967 *Science, N.Y.* **157**, 55–56.

Hide, R. 1970 *Q. Jl R. met. Soc.* **96**, 579–590.

Hide, R. 1977 *Phil. Trans. R. Soc. Lond.* A **284**, 547–554.

Hide, R. & Horai, K. I. 1968 *Physics Earth planet. Inter.* **1**, 305–308.

Hide, R. & Malin, S. R. C. 1970 *Nature, Lond.* **225**, 605–609.

Hide, R. & Malin, S. R. C. 1971*a* *Nature, Lond.* **230**, 63.

Hide, R. & Malin, S. R. C. 1971*b* *Nature, Lond.* **232**, 31–33.

Hide, R. & Malin, S. R. C. 1971*c* *Comments Earth Sci.: Geophys.* **2**, 1–13.

Hide, R. & Malin, S. R. C. 1972 *Trans. Am. geophys. Un.* **53**, 610.

Jones, G. M. 1977 *J. geophys. Res.* **82**, 1703–1709.

Kaula, W. M. 1966 *J. geophys. Res.* **71**, 5303–5314.

Khan, M. A. 1971 *Nature, Lond.* **230**, 57–61.

Lambeck, K. 1980 *The Earth's variable rotation: geophysical causes and consequences* (449 pages.) Cambridge University Press.

Lowes, F. J. 1971 *Nature, Lond.* **230**, 61–62.

Moffatt, H. K. & Dillon, R. F. 1976 *Physics Earth planet. Inter.* **13**, 67–78.

Phillips, R. J. & Lambeck, K. 1980 *Rev. Geophys. Space Phys.* **18**, 27–76.

Rubincam, D. P. 1981 *NASA tech. Man.* no. 83825. (61 pages.)

Discussion

K. LAMBECK (*Laboratoire de Géophysique et Géodynamique Interne, Université Paris-Sud, France*). Malin & Hide postulate that the low-degree harmonics in the gravity field originate from an irregularly shaped core–mantle boundary. This can at best be only approximately true because seismic studies suggest that (i) the length scale of lateral variations in the mantle may be much less than the $2\pi R/l$ suggested by the spherical harmonic expansion (R is the radius of the Earth and l the degree of the harmonic), (ii) such variations occur throughout the mantle (see, for example, Julian & Sengupta 1973; Dziewonski *et al.* 1977), and (iii) there is some evidence to suggest that density anomalies tend to be compensated for by anomalies of opposite sign at some greater depth.

These observations lead to the conclusion that at least part of the low-degree harmonics in the gravitational potential is a consequence of much shorter-wavelength density anomalies in the upper mantle. A simple approach is to consider a model in which the mantle density anomalies are of short wavelength and are distributed both with depth and lateral position. If such anomalies occur throughout the mantle then the theoretical spectrum approximates quite closely the observed spectrum (Lambeck 1976). In particular, more than 50% of the power in the low-degree harmonics ($2 \leqslant l \leqslant 5$) originates from density anomalies above about 400 km depth. Gravity anomalies originating from long-wavelength topographic bumps on the core–mantle boundary will therefore be partly masked by the contributions from these shorter-wavelength density anomalies above the interface.

Another model is one in which gravity anomalies are attributed to a specific mantle structure. Subduction zones provide one example. Two recent attempts at evaluating the gravity due to the density anomalies in the subducted lithosphere are by Crough & Turdy (1980) and McAdoo (1981), and both studies indicate that there may be significant contributions to the low-degree harmonics in the geopotential.

It has been recognized by Hide and his colleagues that the bumps on the interface imply some form of dynamic support and that they are therefore associated with convection in the lower mantle. Hence whether gravity anomalies above the bump should be positive or negative is not immediately obvious: a positive bump (where the core intrudes into the mantle) will be associated with a rising branch of the convection cell and the gravitational attraction of the mass deficit nearer the surface may partly or wholly cancel out the bump effect. Any deformation of the upper surface – or of the 650 km discontinuity if there is no convection across it – will further complicate the gravity anomaly, and it is not obvious whether the final anomaly over the bump will be positive, negative or worse, insignificant.

While these various models are unlikely to reflect the real situation very closely they do all lead to a similar conclusion: that a simple one-to-one relation between the low-degree harmonics of the gravity and magnetic fields should not be expected. Yet the results of Malin & Hide are most suggestive. Perhaps the Earth is, for once, actually simpler than our models.

References

Crough, S. T. & Turdy, D. M. 1980 *Earth planet. Sci. Lett.* **48**, 15–22.
Dziewonski, A. M., Hager, B. M. & O'Connell, R. J. 1977 *J. geophys. Res.* **82**, 239–255.
Julian, B. R. & Sengupta, M. K. 1973 *Nature, Lond.* **242**, 443–447.
Lambeck, K. 1976 *J. geophys. Res.* **81**, 6333–6340.
McAdoo, D. C. 1981 *J. geophys. Res.* **86**, 6073–6090.

R. Hide, F.R.S. I trust that the published version of our paper will provide a satisfactory answer to the questions raised by Professor Lambeck and Dr Lowes as to what properties of the gravitational and magnetic fields should be compared. Dr Malin in the short time at his disposal merely touched upon these points in his talk. They were also mentioned, albeit briefly, in my contribution to the discussion of Dr Haddon's paper, when I explained the geophysical background to the work that led to the discovery that large-scale features of the Earth's gravitational and magnetic field are well correlated. Further theoretical work on the effects of bumps on the core–mantle interface on core motions is needed here and I hope that we shall hear from Professor Moffatt (whose work with Dillon is mentioned in our paper) on this matter.

If positive bumps (i.e. those protruding into the mantle) are associated with rising convection currents in the mantle and negative bumps with sinking convection currents then gravitational effects due to bumps could be well correlated with those due to density inhomogeneities in the mantle but opposite in sign. So far as the ratio of the respective magnitudes of these gravitational effects are concerned, this can be estimated at about 20% or a little less from the work of R. Hide and K.-I. Horai (*Physics Earth planet. Inter.* **1**, 305–308 (1968); see also R. Hide, *J. geophys. Res.* **75**, 2141 (1970)), who attempted to set limits on the size of bumps by calculating *hypothetical* shapes of the core mantle boundary for $n = 4$, 6 and 8 when all the non-axisymmetric features of the Earth's gravitational field up to degree n are taken to be due solely to core–mantle interface bumps. Professor Lambeck, among others, has apparently read more into this work of Hide & Horai than was intended. I have never considered that core–mantle interface topography could be *solely* responsible for the long-wavelength features of the Earth's gravitational field.

H. K. Moffatt (*Cambridge University, U.K.*). I wish to make two comments concerning the very simplified model of flow over a bumpy core–mantle interface that was studied by Moffatt & Dillon (1976). In this model, the only source of both gravitational and magnetic perturbations was the interface topography. Even so, the correlation between the two potentials above the interface, maximized with respect to relative displacement, was still in general far from perfect, the reason being that the shift that is optimum for one Fourier component is not optimum for others. The correlation that we calculated was in fact only about 0.72 using the best estimates of the physical parameters of the core. The fact that the observed correlation (with optimum shift) is as high as 0.8366 is all the more remarkable.

Secondly, the same simple model can be used to estimate the mean tangential stress at the interface, which works out (H. K. Moffatt, *Geophys. Astrophys. Fluid Dyn.* **9**, 279 (1978)) at $0.0018a^2$ N m^{-2}, where a is the r.m.s. bump amplitude in kilometres. If $a \approx 1$ km, this is about one-twentieth of what is generally inferred from studies of the decade fluctuations in the length of day.

R. Hide. In the published version of our paper Dr Malin and I mention the results of studies of very simplified models of flow over a bumpy core mantle interface, including the important work of Moffatt & Dillon to which Professor Moffatt refers. The mean tangential stress due to the presence of bumps at the core–mantle interface will depend not only on the height of the bumps, the relation of the Earth, the density of the core and the speed of core motions, but also, in a subtle way (see R. Hide, *Phil. Trans. R. Soc. Lond.* A **284**, 547–554 (1977)), on the strength and configuration of the magnetic field in the upper reaches of the core. The low value of this stress found in the Moffatt & Dillon model is associated with the assumed field configuration, which is mainly toroidal. Recent work by M. H. A. Hassan & I. A. Eltayeb (*Physics Earth planet. Inter.* **28** (in the press)) confirms that the stress can be very much larger when the assumed magnetic field configuration is more realistic, with poloidal and toroidal components of comparable magnitude near the core–mantle interface (but not necessarily at depth within the core, where the strength of the toroidal part of the field may substantially exceed that of the poloidal part).